Methodology for projecting macroeconomic variables

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ABSTRACT

This study introduces the methodology current employed by the Independent Fiscal Institution (IFI) in the elaboration of macroeconomic projections (GDP, inflation rate, employment level, exchange rate and interest rate) used as inputs in the tools for simulating fiscal variables, extending transparency to the interested public about the mechanics of estimates. The specification adopted in most of the equations seeks to capture the short-term dynamics of the data and ensure desirable properties in theoretical terms in the relationship between variables over a broader time horizon. The process of developing their own simulation tools is a fundamental requirement for the IFIs to build forecasts autonomously.

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I - Introduction

Senate Resolution No. 42 of 2016 establishes as one of the functions of the Independent Fiscal Institution (IFI) the production and dissemination of forecasts of relevant variables for the construction of fiscal and budgetary scenarios (Article 1, item I). The purpose of this study, in this context, is to provide an overview of how projections of macroeconomic variables (GDP, inflation rate, employment, exchange rate and interest rate) that subsidize the fiscal scenarios presented in the Fiscal Follow-up Report (RAF) are made, increasing transparency to the interested public about the mechanics of estimates.

The process of developing its own simulation tools is a fundamental requirement for IFI to build forecasts autonomously from its own considerations and views on the current state and evolution of the economy. Given the complexity and challenges (both theoretical and practical) involved in the construction and maintenance of macroeconometric models, it was decided to build, in a first stage, tools that would formally represent the behavior of the economy and the relationships between the various variables, while ensuring the consistency of these relationships and the possibility of analyzing alternative scenarios around the central forecasts.

In addition to this introduction, the following sections present the structure and specifications of the equations used.

II - Overview of forecasting process

The current methodological structure used in the forecasting process, although more aggregate and objective, shares characteristics and properties with the more sophisticated large-scale models used, for instance, by the UK IFI (Office for Bugdet Reponsibility - OBR)[2], the US IFI (Congressional Budget Office - CBO)[3] and the Portuguese IFI (Conselho das Finanças Públicas - CFP)[4]. The outlook for real GDP growth is determined by the cyclical movements of the components of aggregate demand in the short term and derived by the production function (aggregate supply) approach, equating to the estimate of potential GDP in the medium and long run. In addition, there are similarities in the structure of the equations, with the wide use of error correction models (ECM)[5] that seek to capture short-term dynamics consistent with the data and ensure desirable properties in the relationship between the variables over a broader time horizon.

The model also presents common characteristics in relation to the work already developed for the Brasilian economy, consulted for the elaboration of this study. Reis et al (1999) presents an econometric model intended for medium and long-term projections and economic policy simulation. Muinhos and Alves (2003) describes a medium-scale macroeconomic model. Mattos and Pessoa (2010) presents a preliminary version of the quarterly macroeconomic model of the IBRE/FGV used for the elaboration of scenarios and simulations in the short and medium term.

Each of the blocks presented in Table 1 (aggregate demand, aggregate supply, price determination and monetary policy and labor Market), interdependent, is composed of a set of variables, assumptions, identity relations and equations estimated individually from statistical techniques.

[2] OBR (2013) presents the model used to produce the central forecast, originally developed by Her Majesty´s (HM) Treasury.
[3] See Arnald (2018). CBO´s macro model consists of over 900 variables and about 600 equations.
[4] See Gonçalves e Moreira (2018). CFP´s macro model comprises a total of 137 equations.
[5] Times series econometric technique useful when two or more variables have common trends. The model describes the short-run deviations from equilibrium.

TABLE 1. SUMMARY: METHODOLOGICAL STRUCTURE

Blocks	Variables	Analytical tools
Demand and supply side	**Very short term GDP**	**Nowcast: Mixed-data sampling (MIDAS)**
	Short-term GDP	**Sum of components**
	Household consumption	Error correction model (ECM)
	Government consumption	Exogenous
	Gross fixed capital formation	Error correction model (ECM)
	Inventories	Hold constant as a share of GDP
	Exports of goods and services	Error correction model (ECM)
	(-) Imports of goods and services	Error correction model (ECM)
	Medium and long term GDP	**Production function**
Price determination and monetary policy	IPCA and GDP deflator	Phillips curve
	Exchange rate	Uncovered parity condition
	Interest rate	Taylor rule
Mercado de trabalho	Participation rate	Auto-regressive model
	Employment	Error correction model (ECM)
	Wages	Error correction model (ECM)

Prepared by: IFI.

Once the relations are established and trajectories of each exogenous variable (determined outside the model) are defined, the baseline scenario for the endogenous variables (determined by the exogenous variables) is constructed.

This procedure also enables presenting alternative scenarios around the central forecast. IFI typically presents three scenarios in its reports: baseline, optimistic and pessimistic. The objective of keeping the three scenarios up to date is to highlight the degree of uncertainty that lies around economic forecasts, as well as their dependence on different configurations of the country´s economic situation and the external environment.

Deviations from the central projection are generated from changes – based on assumptions and judgments – in the trajectory of exogenous variables, such as inflation target, country risk premium, population growth, the future evolution of the total factor productivity, commodity prices, world economic growth, the Fed Funds rate and foreign inflation rate. Regarding international economy, the assumptions used by the IFI derive from the scenarios presented by the International Monetary Fund (IMF) and the CBO in their reports.

III - Real GDP growth

In the very short term, the forecast of real GDP is detemined by models that enables to monitor economic activity in real time, using high-frequency indicators for total GDP and its componentes from the perspective of expenditure (with the exception of government consumption).

In the short run, the outlook for real GDP growth is built on the componentes of the expenditure side. The output gap narrows at some point in the projection horizon (idle capacity or excess demand disappears), at which point the projection of potential GDP provides an anchor for the real GDP growth forecast.

The medium and long-term pattern of output growth is determined by potential output (estimate of the trend around which economic activity fluctuates over the business cycle), which is calculated by means of a production function, reflecting the size of the capital stock, the number of hours worked and the total factor productivity (combined productivity of capital and labor).

Briefly, real GDP growth is determined by the interaction between aggregate demand and supply.

III.1 - Very short term: Nowcasting

The forecast of the real change in Gross Domestic Product (GDP) in the very short term – a predictive horizon of up to two quarters ahead of the last release of the Quarterly National Accounts System by the Brazilian Institute of Geography and Statistics (IBGE) – is based on econometric specifications that extract information form a set of monthly macroeconomic indicators. The objective is to find models that fit well with the data and capture the dynamics of economic activity in real time. Because offical data are published with a lag, the IFI uses a series of equation to estimate GDP before the offical data are published (*nowcasting*).

The main set of variables used to predict GDP the next quarter following the last data released is composed of six indicators:

- Central Bank economic activity index (IBC-Br);
- Industrial production index (PIM-PF) - IBGE;
- Broad retail sales (PMC) - IBGE;
- Volume of services (PMS) - IBGE;
- Industry Installed Capacity Usage Level (Nuci) – Fundação Getúlio Vargas (FGV);
- Statistics from the General Registry of Employed and Unemployed Persons (Caged) of the Special Secretariat for Social Security and Labor.

Another set of monthly data is composed of several series, such as:

- Automobile production released by the National Association of Motor Vehicle Manufacturers (Anfavea);
- Sales of automobiles and light commercial vehicles, released by the National Federation of Automotive Vehicle Distribution (Fenabrave);
- Household´s and business confidence and level of uncertainty - FGV;
- Flow of vehicles on toll roads measured by the Brazilian Association of Highway Concessionaires (ABCR);
- Ipea's Fixed Capital Formation Index;
- Volume of exports and imports of good and services.

Besides containing information on the current state of the economy, they are (to a large extent) released earlier than most of the series included in the main set of variables.

The data update, together with the informative content of the carry-over effect, allows the production of estimates for GDP two quarters ahead of the last release.

The methodology used by IFI is MIDAS ("Mixed Data Sampling") models, a forecasting tool that allows the connection of data sampled at different frequencies in the same regression. More specifically, MIDAS use higher frequency indicators (monthly in this case) to forecast a lower frequency variable (quarterly GDP growth). This econometric approach has been widely used in the forecasting literature and can be found, for example, in recent publications of the Bank of England (Anesti et al, 2017) and the Portuguese Public Finance Council (Gonçalves and Moreira, 2018).

The dependent variable (GDP growth) is quarterly, while the regressors are preserved in their original monthly frequency. In addition, auxiliary regressions are also estimated to forecast the following components of expenditure: household consumption, gross fixed capital formation, exports and imports.

In all, around 30 regressions[6] are estimated for aggregate GDP, with the central forecast being obtained through a combination weighted by past performance of each model. The combination of individual results is adopted in order to mitigate the error and ensure greater robustness of the projection.

[6] The following specifications are employed: "Exponential Almon Lag" and U-MIDAS (Unrestricted Mixed Data Sampling).

The procedure is conducted in the *EViews* software and updated whenever new information are released. After obtaining the forecast for real GDP time series without seasonal adjustment, the deseasonalization is done from the parameters that IBGE discloses in the Quarterly National Accounts[7], through the R software.

III.2 - Short-term: Aggregate demand

In the short and medium term, the GDP forecast is obtained by the sum of the components of aggregate demand, which are modelled individually: household consumption (C), government consumption governo (G), gross fixed capital formation (FBCF), changes in inventories (ΔEstoque), exports (X) and imports of goods and services (M), according to the identity:

- $Y_t = C_t + G_t + FBCF_t + \Delta Estoque_t + X_t - M_t$ (1)

The methods used to forecast each component are described below.

III.2.1 - Household consumption

Household consumption is the largest component of GDP (demand side). Its share fluctuated around 63% between 1996 and 2019.

Labor Market and credit market conditions are among the main drivers of private consumption[8]. Increases in the real household disposable income raise consumption, while increases in the real interest rate of the economy (reflecting tightening credit condition) and in the unemployment rate (capturing possible effects of circumstantial savings increments when households face uncertainties in future income flows) have the opposite effect.

The equation below, estimated with quarterly data, has been specified in order to consider that changes in consumption (Δln C), in logarithm, respond to changes in the available extended payroll (Δln msad), in the *ex-ante*[9] real interest rate (Δr) and in the unemployment rate (Δtx_desemp)[10]. The available extended payroll is an indicator that takes into account the labor income (after income tax and social security contributions) and social protection and security benefits.[11]

- $\Delta \ln C_t = \beta_0 + \beta_1 \Delta \ln C_{t-1} + \beta_2(\ln C_{t-1} - \beta_3 \ln msad_{t-1}) + \beta_4 \Delta \ln msad_t + \beta_5 \Delta r_{t-1} + \beta_6 \Delta tx\_desemp_{t-1} + \beta_7 Q1_t + \beta_8 Q2_t + \beta_9 Q3_t + \beta_{10} D_t + \varepsilon_t$ (2)

Where D_t is a variable that controls the influence of recessive periods throughout the sample; the variables $Q1_t$, $Q2_t$ e $Q3_t$ captures seasonal effects; ε_t is the error term in the equation and Δ is a lag operator, i.e., $\Delta \ln C_t = \ln C_t - \ln C_{t-1}$.

The parameters are estimated by an error correction model, appropriated when the variables under analysis exhibit common trends, as in the case of private consumption and disposable income. The parameter β_2 represents the speed of adjustment to equilibrium between the two variables, correcting for temporary deviations in the long-term relationship, expressed by $(\ln C_{t-1} - \beta_3 \ln msad_{t-1})$. The parameter β_3, obtained in the first stage by *Dynamic Ordinary Least Squares* (DOLS), concerns the long-term income elasticity.

III.2.2 - Government consumption

Government consumption is the second largest component of aggregate demand. Its share fluctuated around 19% between 1996 and 2019.

[7] For further information, see "Anexo a) Notas Metodológicas": https://biblioteca.ibge.gov.br/visualizacao/periodicos/2121/cnt_2020_1tri.pdf
[8] See, por example, Schettini, B. P., et al. (2011).
[9] Calculated by the 1-year forward contract rate (SWAP) minus (smooth) inflation expectations for the next 12 months.
[10] The estimation of the available extended payroll and the unemployment rate will be explored in the labor market section.
[11] See Nota Técnica 26 ("Uma medida de renda disponível das famílias"), available at: https://www2.senado.leg.br/bdsf/bitstream/handle/id/553547/NT_26_2019.pdf

The government consumption projection is adjusted to be consistent with the IFI's scenario for Union expenditures. Despite conceptual differences in accounting and scope between the National Accounts series and the National Treasury (STN) fiscal results series, the idea is to preserve the historical difference between government consumption and the composition of primary expenditure (conceptually approximated to government consumption from the National Accounts as classified by the Federal Budget Secretariat – Secretaria de Orçamento Federal - SOF).

According to the classification of the primary government expenditures in terms of the use proposed by the Federal Budget Secretariat (SOF)[12], government consumption (taking into account only the central government) corresponds to the payroll expenses of active servants and the direct expenditures of all the Powers and Public Prosecutors of the Union with their own consumption.

Following this, government consumption in real terms is obtained by dividing nominal expenditure by the corresponding deflator.

III.2.3 - Gross fixed capital formation

The analysis of gross fixed capital formation (GFCF) is drawn from an error correction model that seeks to capture the dynamics of investment by the variation in the capital stock.

The capital stock measure is constructed from the annual series estimated by Ipea[13] based on the perpetual stock method and a temporal disaggregation of times series from annual to the quarterly frequency using the GFCF series as an indicator of quarterly oscillations.

Following OECD (2015) - an error correction model linking the capital stock to the output level - equation 3 is estimated with quarterly data.

- $\Delta \ln estoque\_K_t = \beta_0 + \beta_1 \Delta \ln estoque\_K_{t-1} + \beta_2 (\ln estoque\_K_{t-1} - \beta_3 \ln PIB_{t-1}) + \beta_4 \Delta \ln PIB_{t-1} + \beta_5 \Delta \ln PIB_{t-2} + \beta_6 \ln EPU_{t-1} + \beta_7 r_{t-1} + \beta_8 Q1_t + \beta_9 Q2_t + \beta_{10} Q3_t + \beta_{11} D_t + \varepsilon_t$ (3)

In addition to the accelerator effect (Δln PIB) on investment, the change in the capital stock (Δln estoque_K), in logarithm, in the short run is also influenced by the level of the real interest rate (r) and by the uncertainty of economic policy [14] (ln EPU), considering the fact that in times of greater uncertainty, planned investment decisions tend to be postponed or reversed. In the long run, the growth rate of the capital stock will evolve in line with the growth rate of output, a phenomenon incorporated into equation (3) by the term $(\ln estoque\_K_{t-1} - \beta_3 \ln PIB_{t-1})$.

Variables $Q1_t$, $Q2_t$ e $Q3_t$ are indicators of quarters that capture seasonal effects and D_t is a variable that controls the influence of recessive periods throughout the sample. The parameter β_2 represents the speed of adjustment to equilibrium between the capital stock and GDP, correcting for temporary deviations in the long-term relationship, while β_3 represents the long-term elasticity of the capital stock to GDP.

III.2.4 - Inventories

The change in inventories corresponds to the net change in inventories of finished goods and raw materials used in the production process. It is an element of balance between supply and demand of goods and services. Due to the difficulty in finding an econometric specification that adequately captures its dynamics, the prospective evolution of this component of aggregate demand is obtained from the assumption that its weight in GDP will remain relatively stable - at the level of the last observation - over the forecasting horizon and that its contribution to GDP growth will hover around zero. The fact that, historically, the changes in the share of stocks have not been high and oscillated around 0.3% between 1996 and 2019 contributed to the adoption of these assumptions.

[12] Available at: https://www.gov.br/economia/pt-br/assuntos/planejamento-e-orcamento/orcamento/publicaoes-sobre-orcamento/informacoes-orcamentarias/arquivos/estatisticas-fiscais/8-despesas-primarias-do-governo-central-pela-otica-do-uso
[13] For methodological details see Morandi e Reis (2004): http://www.anpec.org.br/encontro2004/artigos/A04A042.pdf.
[14] Indicator available on a monthly basis for the Brazilian economy: https://www.policyuncertainty.com/index.html

III.2.5 - Net exports

The dynamics of exports and imports of goods and services are analysed separately. Exports are determined by the demand for imports from trading partners, through an indicator that is expressed by the output growth of the main partners weighted by their share in trade with Brazil, and by the real exchange rate (câmbio_real) - used as a measure of competitiveness of Brazilian exports (equation 4).

Import performance, in turn, responds positively to changes in domestic demand for goods and services, expressed as the sum of household consumption, government consumption and investment, and negatively to the real exchange rate - variable that can be interpreted as a relative price measure between domestic and international goods (equation 5).

Equations 4 and 5, shown below, are also estimated by error correction model (ECM). Exports are denoted by X, imports are denoted by M, and the variables Q^1_t, Q^2_t, e Q^3_t have the same meaning as shown above.

- $\Delta \ln X_t = \beta_0 + \beta_1 \Delta \ln X_{t-1} + \beta_2 \Delta \ln X_{t-2} + \beta_3 (\ln X_{t-1} - \beta_5 \ln PIB\_parceiros_{t-1}) + \beta_4 \Delta \ln PIB\_parceiros_t + \beta_5 \Delta \ln câmbio\_real_{t-1} + \beta_6 Q^1_t + \beta_7 Q^2_t + \beta_8 Q^3_t + \varepsilon_t$ (4)

- $\Delta \ln M_t = \beta_0 + \beta_1 \Delta \ln M_{t-1} + \beta_2 (\ln M_{t-1} - \beta_3 \ln dem\_domestica_{t-1}) + \beta_4 \Delta \ln câmbio\_real_{t-1} + \beta_5 Q^1_t + \beta_6 Q^2_t + \beta_7 Q^3_t + \varepsilon_t$ (5)

III.3 - Medium and long term: Aggregate supply

The forecast of real GDP change over the long term is anchored in the calculation of trend or potential GDP (a concept that can be expressed as the maximum output growth that the economy can achieve without generating inflationary pressures), carried out from the production function approach[15,16].

The calculation of potential GDP can be summarised in five steps. The first step corresponds to the specification of a production function that relates capital input (K) and labor (L) to the product level (Y) over a given period of time t. The most common is to admit a set of simplifying assumptions and a Cobb-Douglas format:

- $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$ (6)

Where A is a measure of total factor productivity (TFP) and α e $(1 - \alpha)$ are the respective capital and labor elasticities to output.

The second step corresponds to the construction of the time series corresponding to the estimates of the intermediate variables of the production function (full description in Table 2).

TABLE 2. DESCRIPTION OF DATA SOURCES AND ESTIMATION PROCEDURES FOR INTERMEDIATE VARIABLES

Variable	Description
Y_t	Seasonally adjusted chained series of the GDP volume index: System of Quarterly National Accounts (IBGE).
K_t	A measure of the capital stock corrected by the level of capacity utilization in the manufacturing industry with seasonal adjustment calculated by the FGV. The annual series available at Ipeadata up to 2008 (Net Fixed Capital Stock) was used for the measurement of the capital stock. The update was based on the gross fixed capital formation series and the average depreciation

[15] See Special Study nº4, available at: https://www2.senado.leg.br/bdsf/bitstream/handle/id/536764/EE_04_2018.pdf.
[16] This approach is a variant of the so-called structural methods that impose restrictions on the structure of the economy based on relationships derived from economic theory, as opposed to non-structural methods that usually use statistical filters to estimate trend output.

	rate between 1995 and 2008 (approximately 5.5% per year). The temporal disaggregation of the annual series to the quarterly frequency was constructed by the Denton[17] method using the GFCF series of the Quarterly National Accounts as an indicator of quarterly fluctuations.
L_t	Hours worked of the employed population (i.e. the working age population corrected for labor market participation and unemployment rates). Hours worked: hours actually worked in all jobs, PNAD Contínua, quarter average, with seasonal adjustment. PIA: people aged 14 and over, PNAD Contínua. PEA/PIA: labor market participation rate, PNAD Contínua. Unemployment rate: unemployment rate for people aged 14 and over, PNAD Contínua, with seasonal adjustment.
α	Share of labour income in national income. It was admitted α = 0,4 following Gomes, Pessôa e Veloso (2003).

Source: IBGE, Ipeadata and FGV. Prepared by: IFI.

The third concerns the estimation of total factor productivity (TPF). A logarithmic transformation is applied to the expression of the production function and its terms are rearranged to assume linear format::

- $\ln(A_t) = \ln(Y_t) - \alpha \ln(K_t) - (1 - \alpha)\ln(L_t)$ (7)

which makes it possible to estimate TPF (A) by replacing the intermediate variables of the second stage.

The fourth step is to estimate the trend levels of inputs and total factor productivity. There are several procedures to estimate the trend (or potential) levels of TPF (A^*) and of capital (K^*) e labor (L^*) inputs, the most usual being the application of some type of statistical filter, such as the HP filter proposed by Hodrick and Prescott (1997).

This fourth stage seeks to incorporate some of the recent improvements in the European Commission's methodology (Havik et al, 2014). First, a bivariate model was adopted to estimate the trend of TFP (A^*_t), controlling for cyclical movements with NUCI. The specification of the SUTSE[18] (Seemingly Unrelated Time Series) bivariate model considers smooth trend and common cycle for the quarterly TFP and NUCI series. Another procedure we adapted from the European Commission was the estimation of the reference unemployment rate (Nawru[19]) by the regression model based on the specification of a Phillips curve.

The fifth and final step calculates the final estimate of potential output (Y^*), obtained by replacing the variables estimated in the previous steps in the expression of the production function to such an extent that:

- $Y^*_t = A^*_t K^*_t{}^{\alpha} L^*_t{}^{1-\alpha}$ (8)

One of the main advantages of this approach derives from its theoretical basis in the structural relationship between production factors and product. This allows decomposing the contribution of each factor and productivity to the movements of potential output and therefore provides a simple instrument to analyse the drivers of potential output performance and to plot future scenarios.

Over the forecasting horizon, the reference measures for trend variables are kept at the latest estimated values (according to the equations below). The projection for population dynamics is obtained from IBGE, while the growth rate of trend TFP is defined exogenously from the observation of historical averages.

- Hours worked$^*_t$ = Hours worked$^*_{t-1}$ (9)
- NAWRU$^*_t$ = NAWRU $^*_{t-1}$ (10)

[17] For further details, see: https://journal.r-project.org/archive/2013-2/sax-steiner.pdf

[18] SUTSE models are inserted in the structural models of unobserved components approach, where the series can be described as the sum of unobserved components (trend, cycle, seasonality and irregular component). The statistical treatment is performed by the representation in state space and the estimation of the parameters is conducted by the Kalman filter.

[19] Non-Accelerating Inflation Rate of Unemployment, referred to in the literature by the acronyms Nairu (Non-Accelerating Inflation Rate of Unemployment) ou Nawru (Non-Accelerating Wage Rate of Unemployment).

- $PEA/PIA^*_t = PEA/PIA^*_{t-1}$ (11)
- $Nuci^*_t = Nuci^*_{t-1}$ (12)

On the basis of the framework presented, it is estimated that over the last 20 years (2000-2019) the average growth rate of potential GDP was 2.4% per year. Given the demographic trend (which should make the average growth rate of the labor stock around 0.8% per year), if productivity grows by 0.5% per year (average for the period 2000-2019) and the investment rate of the economy as a proportion of GDP (of 15.4% in 2019) returns to the average of 18% observed over the last 20 years, potential GDP growth is projected at 1.7% per year.

Alternatively, as can be seen in Table 2, if productivity grows again by 1.5% per year (as during the period 2005-2011) and the investment rate of the economy reaches 20%, the expected growth of potential GDP would be 3.2% per year.

TABLE 2. POTENTIAL GDP GROWTH FOR COMBINATIONS OF INVESTMENT RATE AND TFP

		Investment rate (% of GDP)			
		16	17	18	20
TFP growth rate (%)	-0,5	0.3%	0.4%	0.6%	0.9%
	0	0.8%	1.0%	1.1%	1.5%
	0,5	1.4%	1.6%	1.7%	2.1%
	1	2.0%	2.1%	2.3%	2.6%
	1,5	2.6%	2.7%	2.9%	3.2%

Prepared by: IFI.

III.4 - Output gap

The output gap can be interpreted as the difference (percentage) between actual and potential output:

- $H_t = [\left(\frac{Y_t - Y_t^*}{Y_t^*}\right) * 100]\%$ (13)

It is an indicator of cyclical variations in the economy: a positive gap occurs when the economy operates above its trend (or potential) and is an indication that it is overheated and subject to inflationary pressures, as opposed to the negative gap that suggests an economy operating with idle production factors.

The assessment of the cyclical position of the economy (the size of the output gap) is conducted by the IFI using four different approaches: (i) the univariate HP filter[20]; (ii) the multivariate HP filter according to Areosa (2008); (iii) the production function approach, which uses the HP filter to extract the unemployment rate, the participation rate, the NUCI and total factor productivity trends - methodology based on Souza-Júnior and Caetano (2013); and (iv) the production function approach with the methodological recommendations proposed by the European Commission (presented in the previous section).

In fact, there are several methods available to assess the level of idle capacity in the economy. In practical terms, as every method has its advantages and limitations, the central estimation of the output gap, for the purpose of constructing the macroeconomic scenario, comes from the comparison and evaluation of the set of methodologies listed above - a procedure similar to the one adopted by the OBR[21].

IV - Price determination and monetary police

[20] In order to minimize the "end of sample" problem, the standard procedure of extending the historical series is used.
[21] See OBR (2011).

As we will see below, the inflation rate is modeled from a specification of the Phillips curve, the exchange rate behaviour is derived from the uncovered interest rate parity relationship and the nominal interest rate is defined from a Central Bank reaction function (defined by a Taylor rule).

IV.1 - Inflation

IV.1.1 - IPCA

There are several measures of inflation in the Brazilian economy, with distinct characteristics and uses. Among them, the National Broad Consumer Price Index (IPCA), calculated by IBGE, was designed to measure the price variation of a basket of products and services sold at retail, representative of the consumption of households with incomes from 1 to 40 minimum wages. In addition to being used by the Central Bank as a reference for the inflation target system, the IPCA is relevant to the dynamics of public finances because it is the indexer used in the expenditure ceiling rule and also serves as a basis for the evolution of a large part of government spending.

The outlook for the IPCA is conducted by estimating a Phillips curve for free price inflation, linking current inflation (Π^{L}_{t}) with the inertial component of full inflation (Π_{t-1}), future inflation expectations ($E_t\ \Pi_{t+1}$) twelve months ahead (obtained in the Focus-Market Readout), a measure of the activity level (output gap - reflecting the difference between actual and potential GDP) and a pass-through component that seeks to capture the effect of changes in the nominal exchange rate (R$/US$) and external inflation (Π ext), as measured by the Commodity Research Bureau (CRB) commodity index - in view of the relevance of the effects of commodity price fluctuations on domestic inflation dynamics.

The dependent variable in equation 14 is the free price component, since administered prices have a distinct dynamic, usually following contract rules.

- $\Pi_t^{L} = \beta_1\Pi_{t-1} + \beta_2 E_t\Pi_{t+1} + (1- \beta_1 - \beta_2)(\Pi^{ext}_{t-1} + \Delta e_{t-1}) + \beta_4 output\_gap_{t-1} + \varepsilon_t$ (14)

For simplicity, the CRB index is assumed to remain constant over time at the last observed value: $CRB_t = CRB_{t-1}$.

The forecast for administered price inflation, corresponding to 25% of the IPCA, is obtained in the Central Bank's Focus-Market Readout - up to the last available year. For the following periods, for simplicity, the change is assumed to be equal to the inflation target.

The projection for the full IPCA, in the sequence, is constructed from the expression:

- $\Pi_t^{IPCA} = 75\%\ \Pi_t^{L} + 25\%\ \Pi_t^{A}$ (15)

IV.1.2 - GDP Implicit price deflator

The GDP deflator, obtained in the National Accounts, is a broader measure of inflation that reflects the price change of all domestically produced goods and services (not just a specific basket). The monitoring and projection of this price index is relevant for the construction of the nominal GDP projection (according to equation 16). The future path of fiscal indicators expressed in relation to GDP, such as public debt, is also affected by this issue.

- $\text{Nominal GDP}_t = \text{Nominal GDP}_{t-1} * (1 + \Delta\%\ \text{Real GDP}_t) * (1+ \Delta\%\ \text{GDP Deflator}_t)$ (16)

where $\Delta\%\ \text{Real GDP}_t$ is the change in percentage points, i.e. if real GDP grows hypothetically 1% between $t-1$ e t, then $\Delta\%\ \text{Real GDP}_t = 0{,}01$. The same goes for the term $\Delta\%\ \text{GDP Deflator}_t$.

Between 2000 and 2015, according to Annual National Accounts, the rate of change recorded by the GDP deflator (average of 8.1% per year) evolved 1.4 percentage points (p.p.) above that recorded by the IPCA (average of 6.7% per year). The average rate of change in the household consumption deflator (7.6% per year) - the main determinant of the expansion of the GDP deflator - exceeded that recorded by the IPCA (6.7% per year) by 0.9 percentage points, even though they are closer in conceptual terms, since they capture the increase in the cost of living from the consumers' perspective. The differences between the growth rates of the household consumption deflator in the National Accounts

and the consumer price index derive from the effects of the construction of the indices themselves, derived, for example, from different formulas, weights and scope[22].

As an assumption, the average difference of 0.9 percentage point between the rates of change in the household consumption deflator and the IPCA between 2000 and 2015 is assumed to remain in the coming years, the government consumption deflator and the GFCF deflator will grow in line with the IPCA and the export deflator will grow in line with the import deflator (which implies a neutral scenario for the evolution of the terms of trade). Thus, the GDP deflator should evolve 0.57 p.p. (0.9 p.p. multiplied by the average share of household consumption expenditure in GDP) above the IPCA in the coming years, according to the equation:

- $\Delta\%\ \text{Deflator}_t = \Delta\%\ \text{IPCA}_t + 0{,}57\ \text{p.p.}$ (17)

IV.2 - Exchange rate

The behaviour of the exchange rate in the short run is obtained under the assumption that the best estimate for the nominal exchange rate is the last observed value (random walk).

- $e_t = e_{t-1}$ (18)

In the medium term, the behaviour of the exchange rate is derived from the uncovered interest parity condition, which expresses the depreciation of the nominal exchange rate (Δe_t) as a function of the interest differential and the sovereign risk premium. In this configuration, when the interest differential widens - as international interest (i^*_t) exceeds domestic interest (i_t) - and risk rises, the local currency depreciates.

- $\Delta e_t = \beta_0 + \beta_1(\Delta i^*_t - \Delta i_t) + \beta_2 \Delta \text{risk\_premium}_t + \varepsilon_t$ (19)

The forecast for the international interest rate (measured by the US Fed Funds rate - US basic interest rate defined by the Federal Reserve) is obtained within the framework of CBO projections[23], while the trajectory of the country risk (represented by the Embi-Br variable[24]) is arbitrarily defined, based on the assessment and judgment of assumptions about the domestic and international environment.

The real exchange rate, in turn, is obtained by the following identity:

- $\Delta \text{rer}_t = (e_t - e_{t-1}) + \Pi_t - \Pi^*_t$ (20)

Where the change in the real exchange rate Δrer_t is expressed in terms of the change in the nominal exchange rate ($e_t - e_{t-1}$) and the domestic inflation and trading partners differential ($\Pi_t - \Pi^*_t$). The projection for external inflation (approximated by US inflation) is obtained in the economic scenario outlined by the CBO.

IV.3 - Interest rate

The monetary policy decision is approximated by a Taylor rule, an equation that relates the Selic rate (i_t) - the nominal interest rate that is the monetary policy instrument of the Central Bank - with its autoregressive or persistence component ($\rho\ r_{t-1}$), the natural nominal interest rate ($r^n + \Pi^*_t$), the inflation deviation from the target ($\Pi_t - \Pi^*_t$) and the output gap (output_gap_t).

- $i_t = \rho r_{t-1} + (1-\rho)[(r^n + \Pi^*_t) + \phi_\Pi(\Pi_t - \Pi^*_t) + \phi_h\ \text{output\_gap}_t]$ (21)

[22] For more details, see Nota Técnica 18 ("Discussões sobre a evolução do deflator do PIB"), available at: https://www2.senado.leg.br/bdsf/bitstream/handle/id/544424/NT18_2018.pdf?sequence=1
[23] Which can be accessed at: https://www.cbo.gov/about/products/budget-economic-data#4
[24] Emerging Markets Bond Index – Brazil is calculated by JPMorgan, representing, in basis points, the sovereign spread between benchmark country securities and US Treasury securities.

Where ρ represents the coefficient associated with the term autoregressive, ϕ_Π the coefficient associated with the sensitivity of the monetary authority to the inflation gap and ϕ_h the coefficient related to the sensitivity to the output gap.

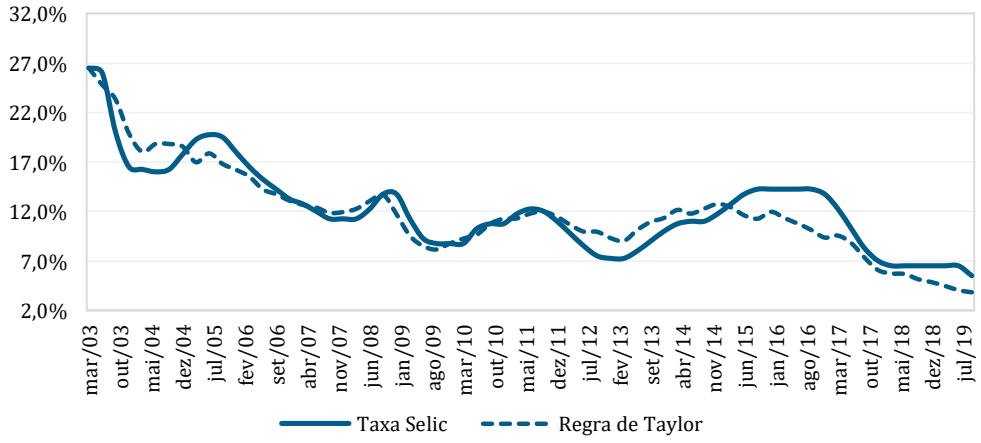
In the long run, when effective GDP is equal to potential GDP (output gap = 0) and the inflation rate is anchored to the target ($\Pi_t = \Pi_t^*$), the estimated value for the Selic rate moves towards the value of the natural nominal interest rate.

Like the output gap, the natural real interest rate is an unobserved variable with great uncertainty associated with its estimation. The variable is, however, a reference point in the conduct of monetary policy under the inflation target regime as it represents the interest rate consistent with GDP growth equal to potential growth and the inflation rate on target. The interest rate, being below the natural rate, stimulates economic activity. When the interest rate is above the natural rate, its effect on output is contractionary. The Central Bank does not disclose its estimates for this variable.

A simplified way of estimating the natural interest rate is to extract the trend real interest rate ex ante by means of statistical filters (such as the HP filter). Alternatively, the natural interest rate can be derived based on the uncovered interest parity condition, where the domestic interest rate equals the international interest rate plus a country risk premium and the expected depreciation of the exchange rate. In this approach, after the sum of the three terms, the HP filter is applied to smooth out fluctuations and extract the trend from the resulting series[25].

Chart 1 illustrates the Selic level prescribed by the Taylor rule by considering a specification that gives equivalent weight to the expected inflation deviation from the target and to the output gap.

Chart 1. Selic rate and Taylor rule prescription



Once the nominal interest rate is set, the real (ex-ante) rate is calculated from the 1-year future rate (360-day pre-DI nominal swap interest rate) discounting expected inflation over the next 12 months. The trajectory of the 360-day swap rate, in turn, depends on the expected Selic rate over the life of the contract and a premium. Following the Central Bank (2017), the cu0072ve for the 360-day pre-DI swap premium is modeled as a function of its lag and the country risk premium (measured by Embi-Br).

- $premium_t = \beta_0 + \beta_1\, premium_{t-1} + \beta_2 risk\_premium_t + \varepsilon_t$ (22)

V - Labor Market

[25] See Fernando de Holanda Barbosa et al (2016).

The unemployment rate is calculated by the number of people who are unemployed (PD) as a proportion of the employed (PO) and unemployed. The denominator (PO + PD) in the identity below expresses the labour force (PEA).

- $TD_t = 100(\frac{\text{PDt}}{\text{POt + PDt}})$ (23)

The number of unemployed (PD) can be expressed as the difference between the labour force and the employed population (PEA – PO), and the labour force (PEA) as the result of the multiplication between the working-age population and the labour force participation rate (PIA x (PEA/PIA)).

In these terms, the projection of the unemployment rate can be obtained from the expected dynamics for the working-age population (PIA), the labour force participation rate (razão PEA/PIA) and the total number of persons employed.

V.1 - Labor supply: labour force participation rate

The perspective for the working-age population (defined by IBGE as the total of people aged 14 and over) is exogenously determined from IBGE demographic projections, while the labour force participation rate is modeled using equation 24, based on Burns et al (2019), on a quarterly basis, under the assumption that the variable follows an auto-regressive process and moves towards its trend level $(PEA/PIA)^*$, obtained through an HP filter over the original series.

- $PEA/PIA_t = \rho\ (PEA/PIA)^*_t + (1-\rho)\ (PEA/PIA)_{t-1}$ (24)

In parallel, the trajectory resulting from this equation is compared with the estimate for the labour force participation rate in the Brazilian labour market published by *International Labour Organization* (ILO)[26], that takes into account its cyclical (general economic and labour market conditions) and structural (such as the degree of flexibility of labour agreements) determinants. The final projection of the PEA/PIA relationship, in turn, is obtained from the judgment derived from the joint analysis of these two approaches.

V.2 - Labor demand: employment

The evolution of the employed population is determined by means of an error correction model, specified in equation 25, through which the dynamics of employment in the short run is determined by changes in real wages (wages) and in economic activity (output gap). In the long run, the employment growth rate converges to the potential growth rate of the labour force(PO^*) through the ratio ($\ln PO_{t-1} - \beta_2 \ln PO^*_{t-1}$).

- $\Delta \ln PO_t = \beta_0 + \beta_1(\ln PO_{t-1} - \beta_2 \ln PO^*_{t-1}) + \beta_3\ \Delta \text{ hiato}_{t-1} + \beta_4\ \Delta \ln \text{salário}_t + \beta_5\ \Delta \ln PO^*_t + \beta_6 Q1_t + \beta_7 Q2_t + \beta_8 Q3_t + \varepsilon_t$ (25)

V.3 - Wages

The projection of the total labour real income is also determined by means of an error correction model, specified in equation 26. The dynamics of real wages in the short run includes the behaviour of inflation (IPCA), labour productivity (produtividade_L) and the gap between the unemployment rate and its trend level (TD - Nawru). In the long run, real wage growth is assumed to occur in line with labour productivity growth – by means of ($\ln \text{salário}_{t-1} - \beta_2 \ln \text{produtividade\_L}^*_{t-1}$).

- $\Delta \ln \text{salário}_t = \beta_0 + \beta_1(\ln \text{salário}_{t-1} - \beta_2 \ln \text{produtividade\_L}^*_{t-1}) + \beta_3\ \Delta \ln IPCA_t + \beta_4\ \Delta \ln \text{produtividade\_L}_t + \beta_5(TD_t - NAWRU_t) + \beta_6 \Delta \ln \text{salário}_{t-1} + \beta_7 Q1_t + \beta_8 Q2_t + \beta_9 Q3_t + \varepsilon_t$ (26)

The overall gross labor income (MBR), in the sequence, is obtained by multiplying the occupied population (PO) with the total labour real income (RM). The available Extended Payroll (MSAD) - net of income tax (IR) and social security contributions (CP) -, the explanatory variable for the household consumption forecast model, is defined as follows:

[26] Available at: https://ilostat.ilo.org/topics/population-and-labour-force/

- $MSAD_t = MSA_t - (IR_t + CP_t)$ (27)
- $MSA_t = MRB_t + BPS_t + BP_t$ (28)
- $MRB_t = RM_t * PO_t$ (29)

In addition to the overall gross labor income (MRB), the expanded indicator takes into account other sources of resources that are configured as income for households. To this end, a set of social protection (BPS) and social security (BP) benefits paid by the federal government in the form of cash transfer policies are incorporated into the construction of the indicator.

VI - Conclusion

This study presented the most recent version of the models used by the IFI to make projections of macroeconomic variables that subsidize the fiscal scenarios presented in the Fiscal Follow-up Report (RAF).

In order to improve the current methodological framework for macro-fiscal scenario analysis and forecasting and to implement improvements for the fulfilment of its mandate, the IFI has constantly monitored the forecasting tools and strategies used, in particular, by the CBO, CBO, CFP and multilateral organizations - such as the World Bank's macroeconomic and fiscal model (MFMOD)[27]-, in addition to the technical assessment work published by the OECD on the adequacy of models and methods adopted by other IFIs around the world.[28]

The next steps mapped out so as to improve the current methodological framework are: the estimation of VAR (Vector Autoregression) type statistical models for forecasting inflation in the short run; separation of GFCF estimation between private and public investments; quantitative analysis of the effect of economic reforms on total factor productivity, through methodologies such as the one used in Égert (2018); endogenization of the sovereign risk premium - to be estimated as a function of fiscal, solvency and liquidity indicators - as in Muinhos, Alves and Riella (2002).

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