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The elasticity of tax revenues to GDP

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ABSTRACT

The change in tax revenues in response to changes in the GDP is one of the main parameters used by the Independent Fiscal Institution (IFI) to develop its macro-fiscal scenarios, regularly published in the Fiscal Follow-Up Reports (FFR). This brief paper updates the IFI revenue elasticity estimates used in such scenarios. Unlike our former analysis, in Technical Note Nº 19, 2018, we estimate different elasticities for negative and positive output gap[1] periods. The results indicate that the long-term elasticity of total revenues is less than unity, while the short-term elasticity is greater than unity, regardless of the output gap sign. Thus, total revenues tend to grow more than 1% in the short run, and less than 1% in the long run, in response to a 1% growth in GDP.

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[i] IFI analysts.

[1] The output gap is the difference between observed and potential GDP. The potential GDP is estimated by the IFI through the production function methodology.

I – Introduction

The tax revenue elasticity to GDP is defined as the **percentage change in tax revenue in response to a 1% change in the Gross Domestic Product (GDP)**. In this calculation, one must disregard the share of the revenue variation resulting from legal changes. Thus, the elasticity measures only the part of the revenue variation that can be attributed to GDP growth (or contraction). There is also another measure of revenue response to GDP in the literature, known as *tax buoyancy*. This measure, not to be considered in this paper, differs from tax *elasticity* precisely because it considers the effect of legal changes on revenues (for example, the increase in a particular tax rate).

This study updates the IFI estimate for the revenue elasticity, first published in 2018 in the Technical Note (TN) Nº 19.[2] The long-term elasticity estimated in that paper was slightly below unity. This time, we let the estimator be more flexible, potentially different in periods of positive and negative output gaps. The new results indicate that long-run elasticities (valid for analyses covering intervals of years, as explained below) are less than unity for revenues measured at more aggregate levels, such as total revenues and net revenues. Short-term elasticities (valid for analyses covering only a few quarters), in contrast, are above unity for these measures, except for net revenues, whose short-term elasticity is also below unity.

According to the Federal Senate Resolution Nº 42 of 2016, one of the IFI attributions is to "publish estimates of parameters and variables relevant to the construction of fiscal and budgetary scenarios" (article 1, item I). The tax revenues elasticities estimated in this paper will be used in the next updates of the IFI's macro-fiscal scenarios, regularly published in the Fiscal Follow-Up Reports (FFR).

It is worth mentioning that, currently, the projections for federal public revenues presented in the FFRs are based on an elasticity equal to one. From now on, we will be able to calibrate those forecasts according to the output gap. Finally, this work is a timely contribution to the current debate on the recovery of tax collection in this post-recession moment.

II – The problem of estimating the revenue elasticity

In principle, it would sound reasonable to assume that the revenue elasticity to GDP is always equal to one since the main component of primary public revenues is taxes levied on the economy's flow of goods and services. As an illustration, consider a simplified setting where revenues in quarter t (denoted by R_t) are equal to a proportion p of the GDP in the same quarter. In this hypothetical case, it would be true that $R_t = p \cdot GDP_t$, and the percentage change in revenues between quarters t and $t+1$ would equal

$$\frac{R_{t+1} - R_t}{R_t} = \frac{p \cdot GDP_{t+1} - p \cdot GDP_t}{p \cdot GDP_t} = \frac{p}{p} \cdot \frac{GDP_{t+1} - GDP_t}{GDP_t} = \frac{GDP_{t+1} - GDP_t}{GDP_t}$$

i.e. the same percentage change in GDP. If the latter increased by 1%, revenues would also increase by 1%.

However, several characteristics of the tax system and the economy can shift the elasticity away from unity in each quarter. Among the tax system characteristics, there are the degree of progressivity and the seasonality of tax collection. Among the characteristics of the economy, there are the sector composition (agriculture, industry, and services) and the

[2] Available at: https://www2.senado.leg.br/bdsf/bitstream/handle/id/545264/NT_Elasticidade.pdf

tax base composition (labor income, financial transactions, etc.). Depending on these characteristics and their changes over time, the elasticity will change from quarter to quarter.[3]

Given the uncertainty about the value this parameter takes at each moment, economists usually resort to statistical techniques to calculate a unique number – a reference value for the elasticity –, for forecasting revenues (once the GDP has been forecasted in a previous step). Such calculation is the aim of this paper.

II.1 – Long-run and short-run elasticities

In the previous subsection, we stated that economists usually estimate a single reference value for the elasticity. One can go a step further and estimate two (rather than only one) reference values: the "long-run" and "short-run" elasticities.

The **long-run** elasticity (which we will denote by LRE) is a fixed value around which the elasticity observed in the economy tends to vary over extended periods (a few years). In turn, the **short-run** elasticity (SRE) is also a fixed value but measures the intensity of revenue change between two consecutive quarters. Thus, the SRE will be useful to forecast revenues in the short term, while the LRE will be useful to forecast revenues over longer time intervals, for example, the years 2023 through 2030.

To provide a background, Chart 1 presents, for each quarter since 1997, the ratio between the percentage change in nominal revenues and the percentage change in nominal GDP, that is, the observed elasticity. The revenue time series used in Chart 1 (and in the estimates) have undergone a process of removing non-recurrent events so that they are measures of *recurrent revenues*.[4] One of the series corresponds to a share of the federal government revenues that must be transferred to subnational governments. This share is formally denominated "transfers by revenue sharing."

Consistent with the explanation in the introduction, Chart 1 shows that the observed elasticity varies over time, following a clear seasonality, attributable primarily to the tax collection legal schedule (a fact that will be taken into account in the statistical models below). The observed elasticity tends to oscillate around positive values between zero and one, but not to the point of becoming persistently close to one. In other words, no time series appears to satisfy $LRE = 1$.

Another apparent feature in Chart 1 is the drop and subsequent rise, between 2014 and 2019 approximately, in the series (i) total revenue, (ii) revenue administered by the Federal Revenue Office (RFB[5]), (iii) transfers by revenue sharing, and (iv) net revenue. That path coincides with the recession beginning in the second quarter of 2014[6] and the following period of economic recovery.

Finally, the time series show extreme values in the fourth quarters of 1998 and 2008, and the first quarter of 2003. In these quarters, the percentage change in GDP, in absolute value, was much lower than usual, being close to zero. Since such change is the denominator when calculating the observed elasticity, the ratio showed those extreme variations. These sample points will also be taken into account in the specification of the statistical models.

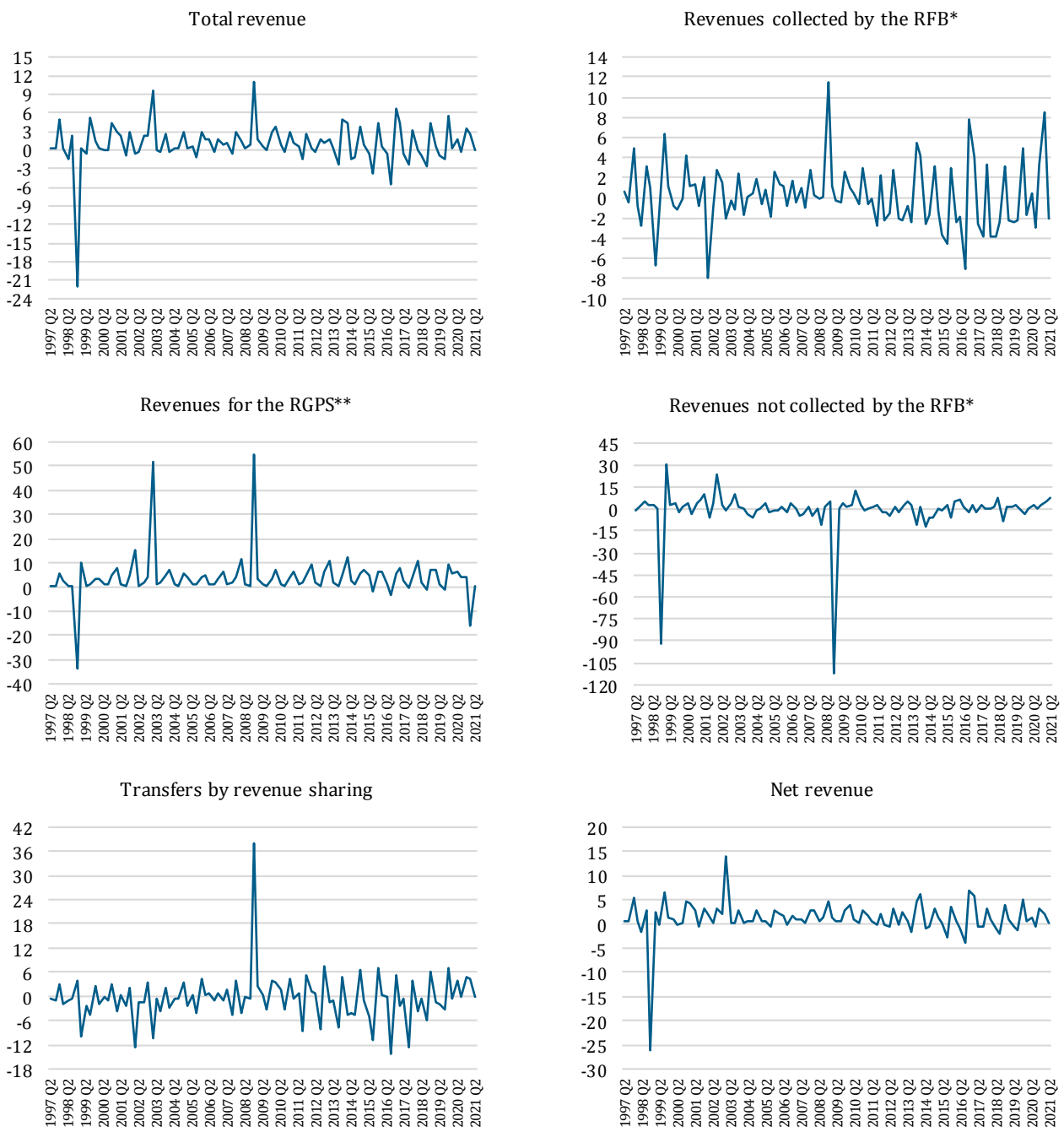
[3] See, for example, Dudine and Jalles (2017) and Gobetti, Orair and Dutra (2018) for a discussion about the factors that can influence tax elasticity and tax buoyancy.

[4] The IFI will publish a study detailing the non-recurrent events considered in this process.

[5] RFB is the Portuguese acronym for the Federal Revenue Office. Its full name is Secretaria Especial da Receita Federal do Brasil.

[6] According to the Business Cycle Dating Committee (CODACE - FGV/IBRE). Available at: https://portalibre.fgv.br/codace

CHART 1. OBSERVED ELASTICITY: RATIO BETWEEN PERCENTAGE CHANGES IN RECURRENT REVENUES AND GDP (NOMINAL DATA NOT SEASONALLY ADJUSTED)

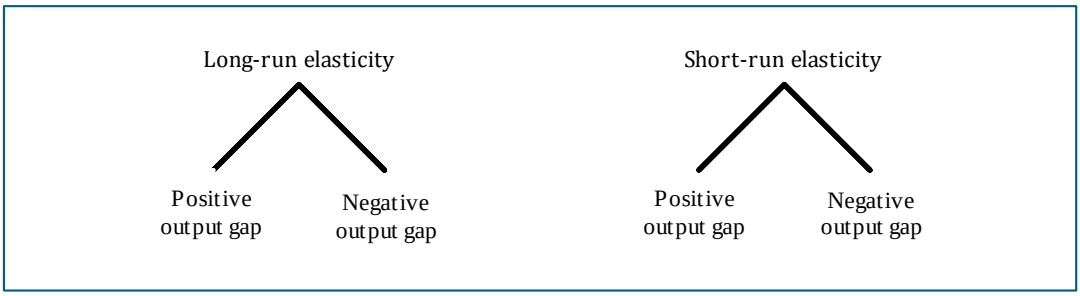


Source: Central Bank (GDP) and IFI (recurring revenues). Prepared by: IFI.
**RFB is the Portuguese acronym for the Federal Revenue Office. Its full name is Secretaria Especial da Receita Federal do Brasil.*
***RGPS is the Portuguese acronym for the social security system covering private-sector employees. Its full name is Regime Geral de Previdência Social.*

II.2 – Elasticities under positive and negative output gaps

In addition to the distinction between long-run and short-run elasticities, we drew a further distinction: elasticities in periods of negative and positive output gaps. Figure 1 shows the complete relationship among the parameters we will estimate. These are four elasticities: LRE under positive output gap, LRE under negative output gap, SRE under positive output gap, and SRE under negative output gap.

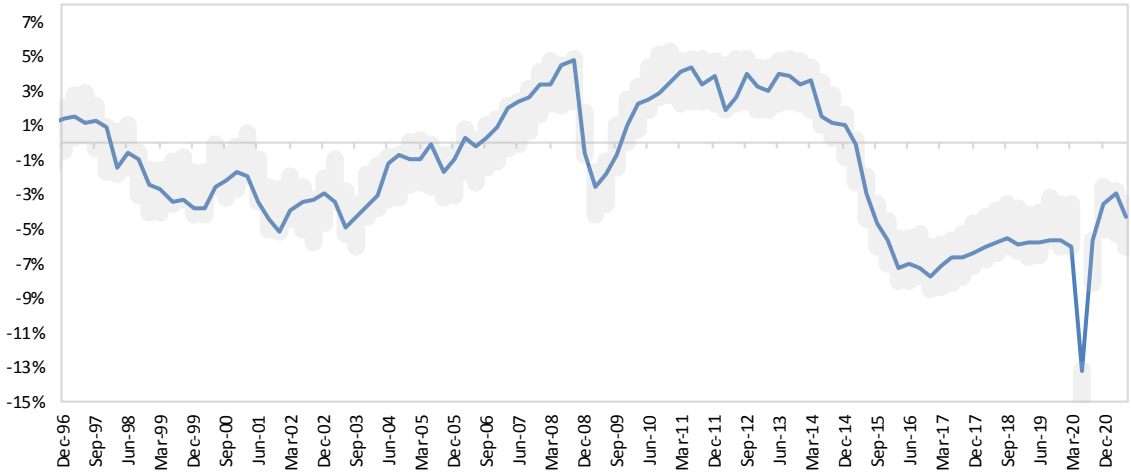
FIGURE 1. RELATIONSHIP AMONG THE FOUR PARAMETERS TO BE ESTIMATED



Prepared by: IFI.

The output gap estimate used to determine if a time interval falls under positive or negative output gap is produced by the IFI. Chart 2 presents the output gap (difference between realized and potential GDP) as a percentage of the potential output per quarter. This estimated time series is regularly updated by the Institution, according to the methodology exposed in the Special Study Nº 4.[7] Note that the output gap has been negative since the second quarter of 2015.

CHART 2. OUTPUT GAP (% OF POTENTIAL OUTPUT) AND PLAUSIBILITY INTERVAL*



*Prepared by: IFI. *Solid line: output gap estimate; shaded area: plausibility interval.*

[7] The study is available at: https://www2.senado.leg.br/bdsf/bitstream/handle/id/536764/EE_04_2018.pdf
The time series is available at: https://www12.senado.leg.br/ifi/dados/arquivos/estimativas-do-hiato-do-produto-ifi/view

III – Methodologies and results

III.1 – Cointegration (long-run elasticity)

The long-run elasticities have been estimated as cointegration parameters using the Dynamic Ordinary Least Squares (DOLS) method.[8] The procedure is standard in the literature, as in Koester and Priesmeier (2017).

We estimated a "log-log" model using quarterly data (1997 Q1 – 2021 Q2),[9] that is, the natural logarithm of the dependent variable (revenue) against the natural logarithm of the explanatory variable (GDP)[10], plus additional controls, such as quarterly dummies due to the seasonality of the series, and dummies to control for outliers visible in Chart 1 (in 1998 Q4, 2003 Q1 and 2008 Q4). One must be cautious about adding controls due to the small sample size and the implied loss of degrees of freedom. However, in the face of some relevant events that occurred over the sample time span, we opted to keep these dummies and their interactions with the GDP.

Model (1) below has been estimated separately for each revenue.

$$\log R_t = \alpha_1 + \alpha_2 D_t^{2008} + \alpha_3 D_t^{gap} + \alpha_4 D_t^{extreme} + \alpha_5 D_t^{covid} + \alpha_6 Q_t^2 + \alpha_7 Q_t^3 + \alpha_8 Q_t^4 \quad (1)$$
$$\beta_1 \log GDP_t + \beta_2 (D_t^{2008} \cdot \log GDP_t) + \beta_3 (D_t^{gap} \cdot \log GDP_t) + \beta_4 (D_t^{extreme} \cdot \log GDP_t) + \beta_5 (D_t^{covid} \cdot \log GDP_t) +$$
$$\gamma_1 \Delta \log GDP_{t-1} + \gamma_2 \Delta \log GDP_{t+1} + \varepsilon_t$$

where R_t is the recurrent revenue or, in one case, transfers by revenue sharing; $D_t^{2008} = 1$ starting in 2008 Q4, and $D_t^{2008} = 0$ otherwise; $D_t^{gap} = 1$ in periods with positive output gap, and $D_t^{gap} = 0$ under negative output gap; $D_t^{covid} = 1$ beginning in 2020 Q1, and $D_t^{covid} = 0$ otherwise; $D_t^{extreme} = 1$ in 1998 Q4, 2003 Q1, and 2008 Q4, and $D_t^{extreme} = 0$ otherwise; Q_t^2, Q_t^3 and Q_t^4 are equal to one in Q2, Q3, and Q4, respectively, and zero otherwise; $\Delta \log GDP_{t-1}$ and $\Delta \log GDP_{t+1}$ are additional controls needed to estimate the cointegration relationship by the DOLS method (we use only one lag and one lead, as in the TN 19[11]).

The role of the interaction $\log GDP_t \cdot D_t^{gap}$ in (1) is to allow for different LREs depending on the sign of the output gap. Since D_t^{gap} is equal to one when the output gap is positive and zero under negative gap, the LRE in the first case is given by $\beta_1 + \beta_3$, while the parameter β_1 itself is the LRE under negative output gap (assuming that the other interactions are set to zero).

[8] Stock and Watson (1993). In a cointegration analysis, it is necessary to ensure that both revenue and GDP time series are integrated of order 1. To this end, we carried out Zivot and Andrews unit root tests on the variables in level (to allow for possible structural breaks) and then, in a second step, augmented Dickey-Fuller tests on the first differences of all series. All revenues and GDP have shown to be integrated of order 1. Another group of necessary tests, the cointegration tests, have been applied on the residuals $\hat{\varepsilon}_t$ of the DOLS implemented through equation (1), to verify their stationarity. We rejected the presence of a unit root in ε_t for all revenues.

[9] We will use the notation Q1, Q2, Q3, and Q4 to refer to the first, second, third, and fourth quarters, respectively.

[10] Mathematically, the difference between the logarithms of a time series, evaluated at two consecutive periods, approximates the percentage change between these periods.

[11] In the TN 19, we used Akaike's criterion to choose the number of leads and lags. Although model (1) is different from the one estimated in TN 19, we opted to keep the same number of lags and leads, also to avoid further loss of degrees of freedom, which would occur either by the addition of controls or by the loss of observations at the beginning and the end of the sample.

Table 1 shows the results. For some revenues, the estimated coefficient is the same in both output gap cases because we set statistically non-significant[12] coefficients to zero. Thus, whenever β_3 was not significant, we considered the elasticity to be the same under either output gap signs.

The results indicate that from a qualitative point of view: (i) the LRE is below unity for the most aggregated revenues, such as total revenue and net revenue, and (ii) when there is a difference between coefficients under distinct signs of the output gap – total revenue, revenues collected by the RFB, and net revenue –, the LRE is larger in **positive** output gap periods than in **negative** output gap periods. Thus, for example, for each 1% growth (contraction) in the GDP, total revenue tends to grow (decrease) by 0.92% when the output gap is negative and 0.98% in periods of positive output gap.

TABLE 1. LONG-RUN ELASTICITIES

Revenue	Positive output gap	Negative output gap
Total revenue	0.98	0.92
Revenues collected by the RFB*	0.98	0.90
Revenues for the RGPS**	1.06	1.06
Revenues not collected by the RFB*	0.71	0.71
Transfers by revenue sharing	1.01	1.01
Net revenue	0.97	0.90

Prepared by: IFI. Note: column "positive output gap" is equal to the sum of estimates $\hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3$, according to equation (1), for each revenue; column "negative output gap" is equal to $\hat{\beta}_1 + \hat{\beta}_2$. For statistically non-significant $\hat{\beta}_i$, $i = 1,2,3$, these $\hat{\beta}_i$ were set to zero.

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It is worth noting that the LRE is a relevant measure of elasticity only for analyses that cover long time intervals (several years). Under atypical circumstances (such as the current Covid-19 pandemic), the response of revenue to GDP may deviate from the pattern in Table 1. In these cases, descriptive and qualitative analyses are an essential complement to econometric estimates.

It is also important to point out that the GDP can undergo **expansions and recessions (cyclical fluctuations)** even within intervals of negative output gap. For example, according to Chart 2, the economy has been under negative output gap since 2015 Q1. After that, Brazil faced a recession until 2016 Q4 and experienced expansion between 2017 Q1 and 2019 Q4, according to the Business Cycle Dating Committee (CODACE - FGV/IBRE).[13] Thus, when we say that revenue responds more to GDP in positive output gap periods than negative ones, this statement holds for both expansions and recessions (and thus negative variations) of the GDP within these periods.

To check the robustness of our results, we considered an alternative model, which includes controls for recession (as dated by CODACE) alongside the other controls already present in (1). However, the coefficients of the variables indicating recessions have not shown to be statistically significant. In another robustness check, we added a deterministic trend to (1), but its coefficient has not shown to be significant either. Such results may be, in part, a consequence of incorporating more controls into a model estimated from a relatively small sample size.

[12] Significance of 10% in a two-tailed test of the hypothesis that the coefficient equals zero, performed with Newey-West standard errors. In this context, statistical significance is a measure, so to speak, of how certain we can be that the estimates (which are computed from samples) are, in fact, different from zero in the real-world economy. The lower the significance, the greater the degree of certainty.

[13] https://portalibre.fgv.br/node/1776

The LRE of total revenue differs from the LREs of other categories. For example, the elasticities of the revenues for the Regime Geral de Previdência Social (RGPS),[14] which are social security revenues, are the largest ones, even larger than unity (1.06), regardless of the sign of the output gap. A possible explanation for this result is that the growth (contraction) of the GDP may be accompanied by a more intense increase (reduction) in the formalization rate in the labor market.[15]

On the other hand, revenues not collected by the RFB showed the lowest LREs (0.71), possibly because they comprise revenues associated with the GDP to a lesser degree, such as revenues from social security contributions of public servants (which are not part of the RGPS system) and revenues from concessions and permissions.

Thus, the higher LRE (revenues for the RGPS) partially offsets the lower LREs (revenues collected and not collected by the RFB), bringing the LRE of the total revenue to an intermediate level (0.98 under positive output gap and 0.92 under negative output gap).

III.2 – Error correction model (short-run elasticity)

T**he short-run elasticities have been estimated using an error correction model (ECM)**, implemented using ordinary least squares, based on equation (2).

$$\Delta \log R_t = \mu_1 + \mu_2 D_t^{2008} + \mu_3 D_t^{gap} + \mu_4 D_t^{extreme} + \mu_5 D_t^{covid} + \mu_6 Q_t^2 + \mu_7 Q_t^3 + \mu_8 Q_t^4 \quad (2)$$
$$\phi_1 \Delta \log GDP_t + \phi_2 (D_t^{2008} \cdot \Delta \log GDP_t) + \phi_3 (D_t^{gap} \cdot \Delta \log GDP_t) + \phi_4 (D_t^{extreme} \cdot \Delta \log GDP_t) + \phi_5 (D_t^{covid} \cdot \Delta \log GDP_t) +$$
$$\omega \Delta \log R_{t-1} + \lambda \cdot ec_{t-1} + u_t$$

where ec_{t-1} is the error correction term (adjusted[16] DOLS residual), lagged by one period, and the other variables are those defined in equation (1). The main differences between equations (1) and (2) are: (i) the changes (first differences)[17] in $\log R_t$ and $\log GDP_t$ are used in place of these variables, and (ii) in equation (2) we included two additional controls, namely the error correction term (ec_{t-1}) and a lag of the dependent variable ($\Delta \log R_{t-1}$), following the procedure of Koester and Priesmeier (2017).

Table 2 presents the results. There are at least one difference and one similarity, concerning Table 1, that are worth mentioning. The difference is that, unlike the LREs, the SREs for total revenue and revenues collected by the RFB are above unity. The similarity is that for those SREs that change with the sign of the output gap – total revenue and revenues for the RGPS – the SRE is larger under **positive** gaps than **negative** ones. Thus, for each 1% growth (contraction) in the GDP, total revenue tends to grow (decrease) by 1.17% when the output gap is negative. When the output gap is positive, total revenue tends to grow (decrease) by 1.51%. The SREs of the revenues not collected by the RFB are not statistically different from zero.

[14] RGPS is the social security system covering private sector employees.

[15] We will evaluate these aspects in more detail in future studies.

[16] As in the TN 19, these residuals are not the $\hat{\varepsilon}_t$ residuals from the DOLS (equation (1)) themselves. We adjusted these residuals before using them as an explanatory variable in the ECM. The adjustment consisted of adding the portions relative to the lags and leads of $\Delta \log GDP_t$ present in (1), so as to obtain: $ec_{t-1} = \hat{\varepsilon}_{t-1} + \hat{\gamma}_1 \Delta \log GDP_{t-2} + \hat{\gamma}_2 \Delta \log GDP_t$.

[17] The first difference of a time series is equal to the difference between its values at two consecutive periods. It is traditionally denoted by the Greek letter Δ. Thus, for a hypothetical time series x_t, its first difference is: $\Delta x_t = x_t - x_{t-1}$. One can prove that, if x_t is equal to the logarithm of another time series z_t (e.g., $x_t = \log z_t$) then $\Delta x_t = \log z_t - \log z_{t-1}$ is an approximation of the percentage change in z_t between periods $t-1$ and t. For this reason, first differences of logarithms are vastly used in the economics literature.

The comparison between Tables 1 and 2 shows that there is no clear pattern involving LREs and SREs. For example, while the LRE of total revenue under positive output gap (0.98) is **lower** than its SRE (1.51), the LRE for the revenue for the RGPS (1.06 under either output gap signs) is higher than its SREs (0.90 and 0.60).

In informal terms, the results in Tables 1 and 2 also show that the relationship between revenue and GDP depends on the economy's circumstances (positive or negative output gap) and the revenue aggregation considered.

TABLE 2. SHORT-RUN ELASTICITIES

Revenue	Positive output gap	Negative output gap
Total revenue	1.51	1.17
Revenues collected by the RFB**	1.02	1.02
Revenues for the RGPS***	0.90	0.60
Revenues not collected by the RFB**	0.00*	0.00*
Transfers by revenue sharing	2.01	2.01
Net revenue	0.74	0.74

Prepared by: IFI. Note: column "positive output gap" is equal to the sum of estimates $\hat{\phi}_1 + \hat{\phi}_2 + \hat{\phi}_3$, according to equation (2), for each revenue; column "negative output gap" is equal to $\hat{\phi}_1 + \hat{\phi}_2$. For statistically non-significant $\hat{\phi}_i$, $i = 1,2,3$, these $\hat{\phi}_i$ were set to zero.

**Statistically non-significant estimates.*

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Unlike the LRE, the SRE is an elasticity measure appropriate only for short-term analyses, in forecasts for revenues a few quarters ahead. In principle, it does not sound reasonable that the response of revenues in the **short run** may differ from the response in the **long run**. After all, the cumulative change in revenues, calculated from the SRE, would become more and more different from the correspondent change calculated using the LRE over time. However, in the short run, it turns out that not only the SRE determines the change in revenue but also the error correction term. Table 3 presents the estimates for the coefficients of the error correction terms – coefficients denoted by λ in equation (2) – for each revenue.

TABLE 3. COEFFICIENTS OF THE ERROR CORRECTION TERMS

Revenue	Coefficient
Total revenue	-0.63
Revenues collected by the RFB**	-0.43
Revenues for the RGPS***	0.00*
Revenues not collected by the RFB**	-0.60
Transfers by revenue sharing	-0.37
Net revenue	-0.74

Prepared by: IFI. Note: the values are the estimates $\hat{\lambda}$, according to equation (2), for each revenue.

**Statistically non-significant estimate.*

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The coefficient of the error correction term is negative because its role is to attenuate, or even reverse, the short-term impact of GDP on revenues whenever the latter deviate from the path compatible with its long-term relationship with the GDP (as described by the LRE). For example, starting from a given quarter t, if the deviation was positive in the immediately preceding quarter ($ec_{t-1} > 0$), the product $\lambda \cdot ec_{t-1}$ in equation (2) is negative, attenuating (or even reversing) the short-term effect of GDP associated with the SRE. Thus, while the LRE is sufficient[18] to forecast revenue variation in long-term analyses, in short-term forecasts, one must combine the SRE with the error correction term, which varies over time.

III.3 – Time-varying coefficients (short-run elasticity)

For comparison purposes, we estimated elasticities using unobservable component models with time-varying parameters, represented in a state-space framework (Kalman filter). To implement the estimation, we used the ElastH[19] package (Figueiredo, 2017), developed by the Economic Policy Secretariat of the Ministry of the Economy (SPE/ME).[20] However, as in Pinto (2018), we estimated elasticities with respect to the observed GDP, not the output gap.

Gobetti et al. (2010) and Schettini et al. (2011) also estimated, via Kalman filter, regressions with time-varying parameters in a state-space framework to assess the elasticities of revenue categories to the GDP in the context of fiscal policy analysis based on the structural primary balance. According to the authors, the presence of nonlinearities in the Brazilian fiscal series justifies the use of this technique.

Algebraically, models represented in state-space form consist of a measurement equation and state equations. In the present work, the measurement equation describes the relationship between the observed series (R_t), the explanatory variable (GDP_t) and the unobservable variables – seasonality (γ_t) and level (μ_t):

$$\log R_t = \phi_t \log GDP_t + \gamma_t + \mu_t + \varepsilon_t$$

The state equations display the dynamics of the unobserved components estimated by the Kalman filter. The level component varies according to a slope term (ν_t). Since the parameter associated with the revenue elasticity to the GDP (ϕ_t) may change over time, an additional state equation is defined to capture its dynamics:

$$\mu_t = \mu_{t-1} + \nu_{t-1} + \xi_t$$
$$\nu_t = \nu_{t-1} + \varsigma_t$$
$$\gamma_t = \gamma_{1,t} + \gamma_{2,t}$$
$$\gamma_{1,t} = -\gamma_{1,t-2} + \omega_{1,t}$$
$$\gamma_{2,t} = -\gamma_{2,t-1} + \omega_{2,t}$$
$$\phi_t = \phi_{t-1} + \eta_t$$

[18] That is, concerning the part of the variation that the statistical model can explain. As mentioned before, there will always exist a share of the revenue that depends on descriptive considerations to be understood, so that a complete account of the revenue path can be provided.

[19] The tutorial for this R package, detailed in Figueiredo (2017), can be found at: https://www.gov.br/fazenda/pt-br/assuntos/politica-fiscal/atuacao-spe/resultado-fiscal-estrutural/codigo-r/elasth.pdf

[20] SPE/ME is the Portuguese acronym for the mentioned secretariat. It full name is Secretaria de Política Econômica do Ministério da Economia.

A set of eight regressions has been estimated for each revenue category, considering different possible combinations for the variances of the level, slope, and seasonality: fixed or time-varying. In addition, automatic identification of interventions, a feature of the ElastH package, has been used to check for the possibility of changes in the level or the slope and the presence of outliers.

The choice of the adequate model for each revenue category was based on tests of homoscedasticity, normality and independence of the residuals.[21] Note, however, that the value of the elasticities under the different assumptions varied very little. Table 4 presents the estimates for one of the revenue categories (total revenue) to illustrate the little variation across models.

TABLE 4. ELASTICITIES BY MODEL
(NOMINAL RECURRENT TOTAL REVENUE X NOMINAL GDP)

Model	Coefficient
1) Stochastic: level, slope, and seasonality	0.850
2) Stochastic: level, slope | Fixed: seasonality	0.851
3) Stochastic: seasonality, level | Fixed: slope	0.851
4) Stochastic: level | Fixed: slope and seasonality	0.860
5) Stochastic: seasonality, slope | Fixed: level	0.846
6) Stochastic: slope | Fixed: level and seasonality	0.853
7) Stochastic: seasonality | Fixed: level, slope	0.851
8) Fixed: level, seasonality and slope	0.851

Prepared by: IFI.

Table 5 below presents the result of the chosen specification for each revenue category. The values refer to the elasticity coefficients at the last sample point.

TABLE 5. ELASTICITIES BY REVENUE CATEGORY

Revenue	Coefficient
Total revenue	0.85
Revenues collected by the RFB*	0.82
Revenues for the RGPS**	0.73
Revenues not collected by the RFB*	0.66
Transfers by revenue sharing	0.70
Net revenue	0.84

Prepared by: IFI.

**RFB is the Portuguese acronym for the Federal Revenue Office. Its full name is Secretaria Especial da Receita Federal do Brasil.*

***RGPS is the Portuguese acronym for the social security system covering private-sector employees. Its full name is Regime Geral de Previdência Social.*

[21] In addition to comparisons of the value of Akaike's information criterion.

We included the output gap and its interaction with GDP in the measurement equation as an additional check. We found no indication, using this methodology, that the elasticities differ between periods of positive and negative output gaps (Table 6). The differences between the two cases are small and statistically non-significant.

TABLE 6. ELASTICITIES AND OUTPUT GAPS

Revenue	Positive output gap*	Negative output gap*
Total revenue	0.859	0.847
Revenues collected by the RFB**	0.825	0.816
Revenues for the RGPS***	0.749	0.739
Revenues not collected by the RFB**	0.653	0.653
Transfers by revenue sharing	0.705	0.700
Net revenue	0.855	0.836

*Prepared by: IFI. *Note: the coefficient of the interaction between output gap and GDP has not shown to be statistically significant for all revenue categories.*

***RFB is the Portuguese acronym for the Federal Revenue Office. Its full name is Secretaria Especial da Receita Federal do Brasil.*

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Overall, the estimates presented in Tables 5 and 6, obtained from the state-space representation using the specifications detailed above, indicate that the short-run elasticities of revenues to GDP are around 0.85. The coefficients have also been estimated using the same variables but at monthly frequency. The results remained virtually unchanged.

The results in this subsection contrast with the SREs, above unity, obtained using the error correction model (Table 2), which did not allow coefficients to vary over time. Besides this difference in magnitude, another difference is that, in the current approach, the SREs have not shown to be sensitive to the sign of the output gap. Therefore, as a parsimonious conclusion, we can say that the time-varying coefficients methodology suggests that revenues response to GDP tends to be moderate in the near future.

IV – Conclusion

The results from the cointegration and error correction analyses provide support for two conclusions.

The first one is that the long-run elasticities (LRE) are below unity for the most aggregate measures of revenues (total revenue, revenues collected by the RFB, and net revenue). The short-run elasticities (SRE), on the contrary, are above unity for these measures, except for net revenue, whose SRE is smaller than unity. These results suggest that, even though there may be intense increases or losses of revenues in the short term (that is, when we consider a few consecutive quarters), the response of revenues to the GDP tends to return to moderate levels in the long run, growing even at a slightly slower rate than the GDP. In other words, for each 1% growth (contraction) in the GDP, total tax revenues tend to grow by less than 1% when we consider longer terms.

The second conclusion is that when LREs and SREs differ between positive and negative output gap periods, they are larger during positive output gaps.

In informal terms, the results show that the relationship between tax revenues and GDP depends on the economy's circumstances (positive or negative output gap). Besides, each revenue category responds in a particular way to the sign of the output gap.

For comparison purposes, we also estimated SREs through another methodology, based on the Kalman filter, and implemented using the ElastH package of the SPE/ME. These elasticities turned out to be less than unity, unlike the SREs estimated using the main methodology of this study (error correction models). Furthermore, there was no statistically significant difference between the estimates under positive and negative output gaps, contrasting with the estimates obtained through the error correction models. Thus, as a parsimonious conclusion, this methodology (Kalman filter) suggests that the response of revenues to GDP tends to be moderate in the near future.

It is important to note that statistical models allow us to forecast only the evolution of the revenue's systematic (i.e., regular, persistent) component. As such, those models do not constitute *per se* the only information needed for the revenue forecasts produced by the IFI. These are obtained by combining the systematic part with the atypical components, which are not detected by the models, but instead measured by the IFI based on the fiscal environment and incorporating, for example, the discussion of budget laws and previously unforeseen events with fiscal impact.

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